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**2009**

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## Part I

### Question: 1

[3 × 9 = 27]

- i. If  $M(\theta) = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ , Show that  $M(x) + M(y) = M(x+y)$ . \*\*

**Answer:**

$$M(\theta) = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$\begin{aligned} M(x) \cdot M(y) &= \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \cdot \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix} \\ &= \begin{bmatrix} \cos x \cos y - \sin x \sin y & \cos x \sin y + \sin x \cos y \\ -(\sin x \cos y + \cos x \sin y) & \cos x \cos y - \sin x \sin y \end{bmatrix} \\ &= \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} \\ &= M(x+y) \end{aligned}$$

- ii. The lines  $x - 2y + 6 = 0$  and  $2x - y - 10 = 0$  intersect at the point A. Find the equation of the line making an angle 45 degree with the positive direction of the x-axis and passing through the point A. \*\*
- iii. Find the equations of the tangents to parabola  $y^2 + 12x = 0$  from the point (3, 8). \*\*
- iv. Find the derivative of  $\sin x^2$  with respect to  $x^3$ .

**Answer:**

$$\frac{d(\sin x^2)}{d(x^3)} = \frac{\frac{dy}{dx}(\sin x^2)}{\frac{d}{dx}(x^3)} = \frac{2x \cos x^2}{3x^2}$$

- v. Evaluate the following integral :  $\int \frac{e^{2x}}{2 + e^x} dx$

**Answer:**

$$\int \frac{e^{2x}}{2 + e^x} dx$$

$$\text{Let } e^x + 2 = t$$

$$\Rightarrow e^x dx = dt$$

$$\therefore \int \frac{(t-2)}{t} dt = t - 2 \log |t| + c$$

$$= e^x - 2 \log |e^x + 2| + c$$

- vi. Evaluate the following limit :  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \tan x}{\cos 2x}$

**Answer:**



$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \tan x}{\cos 2x}$$

Differentiating  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{-\sec^2 x}{-2 \sin 2x} = \frac{-2}{-2 \cdot 1} = 1$

vii. Two horses are considered for a race. The probability of selection of the first horse is  $\frac{1}{4}$  and that of the second is  $\frac{1}{3}$ . What is the probability that :

a. both of them will be selected.

**Answer:**

$$\frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$$

b. only one of them will be selected

**Answer:**

$$\frac{1}{4} \cdot \frac{2}{3} + \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{6} + \frac{1}{4} = \frac{5}{12}$$

c. none of them will be selected.

**Answer:**

$$\frac{3}{4} \cdot \frac{2}{3} = \frac{1}{2}$$

viii. If  $\vec{a}, \vec{b}, \vec{c}$ , are three vectors, show that:  $(\vec{a} + \vec{b}) \cdot \{(\vec{b} + \vec{c}) \times (\vec{c} + \vec{a})\} = 2[\vec{a} \vec{b} \vec{c}]$  \*\*

ix. If  $(-2 + \sqrt{-3})(-3 + 2\sqrt{-3}) = a + ib$  find the real numbers  $a$  and  $b$ . With these values of  $a$  and  $b$ , also find the modulus of  $a + ib$ .

**Answer:**

$$(-2 + \sqrt{3}i)(-3 + 2\sqrt{3}i) = a + ib$$

$$6 - 6 - 7\sqrt{3}i = a + ib$$

$$\Rightarrow a = 0, b = -7\sqrt{3}$$

$$|a + ib| = 7\sqrt{3}$$

x. Solve the following differential equation :  $(x \cos y)dy = ex(x \log x + 1)dx$  \*\*

**Answer:**

$$\int \cos y dy = \int \frac{e^x(\log x + 1)}{x} dx$$

Integrating  $\sin y = \int ex \left( \log x + \frac{1}{x} \right) dx$

$$\sin y = ex \log x + c$$



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**Section A** (Question numbers 2 to 7)**Part II****Question: 2**

[4+3=7]

- a. By using properties of determinants, prove that:

$$\begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix} = 2 + 4\sin 2x$$

**Answer:**

$$\begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$$

$$C_1 \rightarrow C_1 + C_2$$

$$\begin{vmatrix} 2 & \cos^2 x & 4\sin^2 x \\ 2 & 1+\cos^2 x & 4\sin^2 x \\ 1 & \cos^2 x & 1+4\sin^2 x \end{vmatrix}$$

$$R_1 \rightarrow R_1 - R_2 \rightarrow R_2 \rightarrow 2R_3$$

$$\Rightarrow 2 + 4\sin 2x$$

- b. Solve the following linear equations using the matrix method :

$$\begin{aligned} x+y+z &= 9 \\ 2x+5y+7z &= 52 \\ 2x+y-z &= 0 \end{aligned}$$

**Answer:**

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$A^{-1} = -\frac{1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= -\frac{1}{4} \begin{bmatrix} -12 & 2 & 2 \\ 16 & -3 & -5 \\ -8 & 1 & 3 \end{bmatrix} \begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix} = -\frac{1}{4} \begin{bmatrix} -4 \\ -12 \\ -20 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$$x = 1, y = 3, z = 5$$

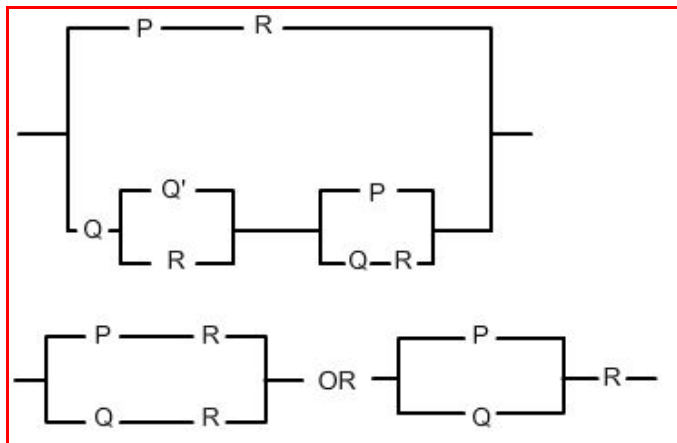
**Question: 3**

[5+5=10]



- a. Prove that the following equation represents a pair of straight lines. Find their point of intersection and the angle between them :  $2x^2 + 7xy + 3y^2 + 2(4x + 7y + 4) = 0$  \*\*
- b. P, Q and R represent switches in 'on' positions and  $P^1$ ,  $Q^1$  and  $R^1$  represent switches in 'off' positions. Construct a switching circuit representing the polynomial  $PR + Q(Q^1 + R) (P + QR)$ . Use Boolean algebra to prove that the above circuit can be simplified to an expression in which, when P and R are 'on' or Q and R 'on', the light is on. Construct an equivalent circuit.

**Answer:**



$$\begin{aligned}
 & PR + Q(Q' + R)(P + QR) \\
 &= PR + (Q.Q' + QR)(P + QR) \\
 &= PR + (0 + QR)(P + QR) \\
 &\quad [\because QQ' = 0] \\
 &= PR + PQR + QR \quad [\because QQR = 0] \\
 &= PR + (P+1)QR \quad [\because P+1 = 1] \\
 &= PR + QR = (P+Q)R
 \end{aligned}$$

**Question: 4**

[5+5=10]

- a. If  $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$  prove that :  $x^2 - y^2 - z^2 + 2yz\sqrt{1-x^2} = 0$

**Answer:**

$$\begin{aligned}
 & \sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi \\
 & \sin^{-1}x + \sin^{-1}y = \pi - \sin^{-1}z \\
 & \sin^{-1}[x\sqrt{1-y^2} + y\sqrt{1-x^2}] = \pi - \sin^{-1}z \\
 & (x\sqrt{1-y^2} + y\sqrt{1-x^2}) = z \\
 & x\sqrt{1-y^2} = z - y\sqrt{1-x^2} \\
 & \text{On squaring we get} \\
 & x^2(1-y^2) = z^2 + y^2(1-x^2) - 2yz\sqrt{1-x^2}
 \end{aligned}$$

- b. Using a suitable substitution find the derivative of  $\tan^{-1} \frac{4\sqrt{x}}{1-4x}$  with respect to x. \*\*

**Answer:**

$$\text{Let } y \text{ be equal to } \tan^{-1} \frac{4\sqrt{x}}{1-4x}$$



$$\therefore y = \tan^{-1} \frac{4\sqrt{x}}{1-4x}$$

$$\text{Let } 2\sqrt{x} = \tan \theta$$

$$y = \tan^{-1} \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$= \tan^{-1} \tan 2\theta$$

$$= 2\theta = 2 \tan^{-1} (2\sqrt{x})$$

$$\frac{dy}{dx} = \frac{2}{\sqrt{x}(1+4x)}$$

**Question: 5**

[4+3=7]

- a. It is given that Rolle's theorem holds good for the function  $f(x) = x^3 + ax^2 + bx$ ,  $x \in [1,2]$  at the point  $x = \frac{4}{3}$ . Find the values of  $a$  and  $b$ .

**Answer:**

$$f(x) = x^3 + ax^2 + bx, x \in [1,2]$$

$$f(1) = a+b+1 = 8 \quad 4a+2b = f(2)$$

$$\Rightarrow 3a + b + 7 = 0$$

$$f'\left(\frac{4}{3}\right) = 3\left(\frac{4}{3}\right)^2 + 2a\left(\frac{4}{3}\right) + b = 0$$

$$\Rightarrow \frac{16}{3} + \frac{8a}{3} + b = 0$$

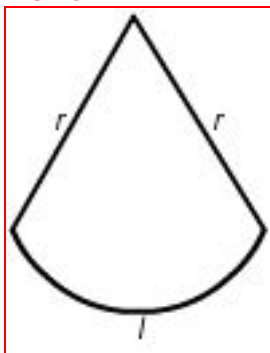
$$\Rightarrow 8a + 3b + 16 = 0$$

Solving (i) and (ii) we get

$$a = -5, b = 8$$

- b. A wire of length 20 m is available to fence off a flower bed in the form of a sector of a circle. What must be the radius of the circle, if we wish to have a flower bed with the greatest possible area ?

**Answer:**



$$2r + l = 20 \text{ m}$$

$$\text{Area} = \frac{1}{2} rl$$

$$A = \frac{1}{2} r(20-2r) = 10r - r^2$$



$$\frac{dA}{dr} = 10 - 2r = 0$$

$$r = 5$$

$$\frac{d^2A}{dr^2} = -2 \text{ (negative)}$$

$\therefore A$  is maximum at  $r = 5$  m

**Question: 6**

[5+5=10]

a. i. Evaluate:  $\int_0^{\pi/2} \log(\tan x) dx$ .

**Answer:**

$$I = \int_0^{\frac{\pi}{2}} \log(\tan x)$$

$$\int_0^{\frac{\pi}{2}} \log \left[ \tan \left( \frac{\pi}{2} - x \right) \right] dx$$

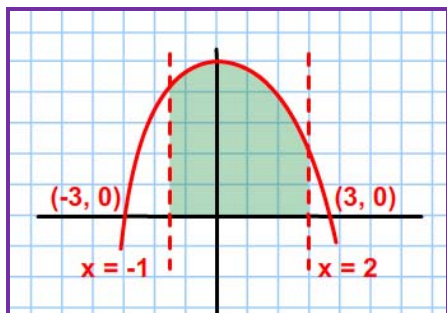
$$\therefore = \int_0^{\frac{\pi}{2}} \log(\cot x) dx$$

$$2I = \int_0^{\frac{\pi}{2}} \log(\tan x \cdot \cot x) dx = \int_0^{\frac{\pi}{2}} \log 1 \cdot dx = 0$$

$$\therefore I = 0$$

ii. Evaluate  $\int_0^5 \left( x + \frac{1}{2} \right) dx$  as a limit of a sum. \*\*

b. Draw a rough sketch of the curve  $x^2 + y = 9$  and find the area enclosed by the curve, the x-axis and the lines  $x + 1 = 0$  and  $x - 2 = 0$ .



**Answer:**

$$x^2 + y = 9 \Rightarrow x^2 = -(y - 9)$$

$$\text{Required Area} = \int_{-1}^2 (9 - x^2) dx$$



$$\left[ 9x - \frac{x^3}{3} \right]_{-1}^2$$

$$= \left[ \left( 18 - \frac{8}{3} \right) - \left( -9 + \frac{1}{3} \right) \right]$$

$$= [27 - 3]$$

24 sq. units

**Question: 7**

[5+5=10]

- a. An examination of 8 applicants for a clerical post was taken by a firm. The marks obtained by the applicants in the Reasoning and Aptitude tests and given below:

Applicant	A	B	C	D	E	F	G	H
Reasoning Test	20	28	15	60	40	80	20	12
Aptitude Test	30	50	40	20	10	60	30	30

Calculate the Spearman's coefficient of rank correlation from the data given above.

**Answer:**

Reasoning test	Aptitude test	R <sub>1</sub>	R <sub>2</sub>	D (R <sub>1</sub> - R <sub>2</sub> )	D <sup>2</sup>
20	30	5.5	5	.5	.25
28	50	4	2	2	4
15	40	7	3	4	16
60	20	2	7	-5	25
40	10	3	8	-5	25
80	60	1	1	0	0
20	30	5.5	5	.5	.25
12	30	8	5	3	9

$$R = 1 - \frac{6[\sum D^2 + \frac{1}{12}(m^3 - m) + \frac{1}{12}(m^3 - m)]}{n(n^2 - 1)}$$

$$= 1 - \frac{6[79.5 + \frac{6}{12} + \frac{24}{12}]}{8 \times 63} = 1 - \frac{6[82]}{8 \times 63}$$

$$= .02$$

- b. Find the value of 'k' so that the second degree equation  $12x^2 - 10xy + ky^2 + 14x - 5y + 2 = 0$  may represent a pair of straight lines. For this value of 'k' find the angle between the lines represented by the above equation.

**Answer:**

$$4x - 5y + 33 \quad \text{and} \quad 20x - 9y - 107 = 0$$

$$\Rightarrow y = \frac{4}{5}x + \frac{33}{5} \quad \dots(i) \quad \text{and} \quad x = \frac{9}{20}y + \frac{107}{20} \quad \dots(ii)$$

- i. Solving both equations we get





$$\bar{x}=13, \bar{y}=17$$

ii. Second line is line of x or y

$$\therefore x = \frac{9}{20} \times 7 + \frac{107}{20}$$

$$= \frac{107}{20} = 8.5$$

iii.  $b_{yx} = r \frac{\sigma_y}{\sigma_x}$

$$\Rightarrow \frac{4}{5} = .6 \times \frac{\sigma_y}{3}$$

$$\Rightarrow \frac{.8}{.2} = \sigma_y$$

$$\Rightarrow \sigma_y = 4$$

**Question: 8**

[5+5=10]

- a. Bag A contains 5 white and 4 black balls, and bag B contains 7 white and 6 black balls. One ball is drawn from the bag A and without noticing its colour, is put in the bag B. If a ball is then drawn from bag B, find the probability that it is black in colour. [5]

**Answer:**

Bag A (5w, 4B) Bag(7W, 6B)

Required Probability =(First white X Second black)+(First black X Second black)

$$= \frac{5}{9} \times \frac{6}{14} + \frac{4}{9} \times \frac{7}{14}$$

$$= \frac{30+28}{126} = \frac{58}{126} = \frac{29}{63}$$

- b. An article manufactured by a company consists of two parts A and B. In the process of manufacture of part A, 9 out of 104 parts may be defective. Similarly, 5 out of 100 are likely to be defective in the manufacture of part B. Calculate the probability that the article manufactured will not be defective. \*\* [5]

**Answer:**

Article will not be defective if both the part are non defective.

$$\therefore \text{Required Probability} = \frac{95}{104} \times \frac{95}{100} = \frac{95}{104} \times \frac{19}{20} = \frac{361}{416}$$

**Question: 9**

- a. Solve the following system of equations using matrices:

$$x + y + z = 6$$

$$x - y + z = 2$$

$$2x + y - z = 1$$

[5]



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**Answer:**

$$z = \frac{13-5i}{4-9i} = \frac{97+97i}{97} = 1+i$$

$$= \sqrt{2} \left[ \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i \right]$$

$$= \sqrt{2} \left[ \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right]$$

$$\therefore z^6 = 8 \left[ \cos \frac{6\pi}{4} + i \sin \frac{6\pi}{4} \right]$$

$$= 8[0-i] = -8i$$

$$dy = (5x-4y)dx$$

- a. Solve the following differential equation for a particular solution.  $dy = (5x - 4y)dx$  , when  $y=0$  and  $x = 0$ . \*\*



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**Section B** (Question numbers 10 to 14)

**Question: 10**

[5+2=7]

- a. Find the equation of the plane which contains the line  $\frac{(x-1)}{2} = \frac{(y+1)}{(-1)} = \frac{(z-3)}{4}$  perpendicular to the plane  $x + 2y + z = 12$ .

**Answer:**

Equation of plane passing through (1,-1,3) is

$$A(x-1)+B(y+1)+C(z-3)=0$$

This is  $\perp$  to line and plane given

$$\therefore 2A-B+4C=0 \text{ and Eliminating A, B and C form equations we get } \begin{vmatrix} x-1 & y+1 & z-3 \\ 2 & -1 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$\therefore (x-1)(-9)-(y+1)(-2)+(z-3)5=0$$
$$-9x+2y+5z-4=0$$

- b. Find the equation of the sphere which passes through the circle  $x^2 + y^2 + z^2 - 6z - 4 = 0$ ,  $x + 2y + 2z = 0$  and whose centre lies on the plane  $2x-y+z=1$ . \*\*

**Question: 11**

[5+3=8]

- a. Find the area of a parallelogram whose diagonals are determined by the vectors  $\underline{a} = 3i+j-2k$  and  $\underline{b} = i-3j+4k$ .

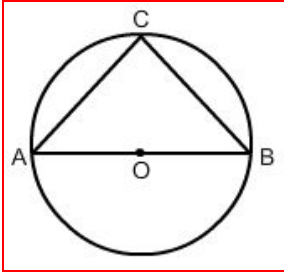
**Answer:**

$$\begin{aligned} \text{Area} &= \frac{1}{2} |\underline{d}_1 \times \underline{d}_2| \\ &= \frac{1}{2} \begin{vmatrix} i & j & k \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} \\ &= \frac{1}{2} [i(4-6) - j(12+2) + k(-9-1)] \\ &= \frac{1}{2} |(-2i - 14j - 10k)| \\ &= \frac{1}{2} (\sqrt{4+196+100}) \\ &= \frac{1}{2} (\sqrt{300}) \\ &= 5\sqrt{3} \text{ sq. units.} \end{aligned}$$

- b. i. Prove by vector method that the diameter of a circle will subtend a right angle at a point on its circumference.



**Answer:**



Let  $\vec{OB} = \vec{b}$  and  $\vec{OA} = -\vec{b}$

And  $\vec{OC} = \vec{c}$

Now  $\vec{CA} = -\vec{b} - \vec{c}$

And  $\vec{CB} = \vec{b} - \vec{c}$

$$\vec{CA} \cdot \vec{CB} = -(\vec{b} + \vec{c}) \cdot (\vec{b} - \vec{c})$$

$$= -\{|\vec{b}|^2 - |\vec{c}|^2\} = 0 \quad [Q |\vec{b}| = |\vec{c}| \text{ radii}]$$

$$\angle ABC = 90^\circ$$

ii. If  $\underline{a}$ ,  $\underline{b}$  and  $\underline{c}$  represent the position vectors of the points with co-ordinates (2, -10, 2), (3, 1, 2) and (2, 1, 3) respectively, find the value of  $\underline{a} \times (\underline{b} \times \underline{c})$ .

**Answer:**

$$\underline{a} \times (\underline{b} \times \underline{c}) = (2\mathbf{i} - 10\mathbf{j} + 2\mathbf{k}) \times \{(3\mathbf{i} + \mathbf{j} + 2\mathbf{k}) \times (2\mathbf{i} + \mathbf{j} + 3\mathbf{k})\}$$

$$(2\mathbf{i} - 10\mathbf{j} + 2\mathbf{k}) \times (\mathbf{i} - 5\mathbf{j} + \mathbf{k}) = 0$$

**Question: 12**

[5+5=10]

a. The mean and variance of a binomial distribution are 4 and 2 respectively. Find the probability of at least 6 successes.

**Answer:**

$$np=4, npq=2$$

$$\Rightarrow q = \frac{1}{2}, p = \frac{1}{2}$$

$$\Rightarrow n = 8$$

$\therefore$  Required probability

$$= {}^8C_6 \left(\frac{1}{2}\right)^8 + {}^8C_7 \left(\frac{1}{2}\right)^8 + {}^8C_8 \left(\frac{1}{2}\right)^8$$

$$= \left(\frac{1}{2}\right)^8 [28 + 8 + 1]$$

$$= \frac{37}{256}$$

b. An insurance company insured 4000 doctors, 8000 teachers and 12000 engineers. The probabilities of a doctor, a teacher and an engineer dying before the age of 58 years are



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0.01, 0.03 and 0.05 respectively. If one of the insured persons dies before the age of 58 years, find the probability that he is a doctor.

**Answer:**

Required probability

$$\begin{aligned} P(\text{Doctor/dies before 58}) &= \frac{P\left(\frac{\text{dies}}{D}\right) \cdot P(D)}{P\left(\frac{\text{dies}}{D}\right) \cdot P(D) + P\left(\frac{\text{dies}}{T}\right) \cdot P(T) + P\left(\frac{\text{dies}}{E}\right) \cdot P(E)} \\ &= \frac{.01 \times \frac{1}{6}}{.01 \times \frac{1}{6} + .03 \times \frac{1}{3} + .05 \times \frac{1}{2}} \\ &= \frac{\frac{.01}{6}}{\frac{.01 + .06 + .15}{6}} = \frac{.01}{.22} \\ &= \frac{1}{22} \end{aligned}$$



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### Section C (Question numbers 13 to 15)

#### Question 13

[5+5=10]

- a. A bill for Rs 84150 is drawn on 22nd April, 2002 at 11 months and is discounted on 11th January 2003. Find the banker's gain if the rate of interest is 10%.

**Answer:**

Amt.=Rs.84,150, Rate =10%=.1

Legally due date= 25 March 2003

Discounted on 11 Jan,2003.

$$\therefore \text{Remaining period} = 20 + 28 + 25 = 73 \text{ days} = \frac{73}{365} = \frac{1}{5} \text{ year}$$

$$\therefore \text{B.D.} = \text{Ani} = 84150 \times \frac{1}{5} \times .1 = \text{Rs.} 1683$$

$$\text{T.D.} = \frac{\text{Ani}}{1 + ni} = \frac{1683}{1 + \frac{.1}{5}} = \frac{1683 \times 50}{51} = \text{Rs.} 1650$$

$$\therefore \text{B.G.} = \text{B.D.} - \text{T.D.}$$

$$= 1683 - 1650$$

$$= \text{Rs.} 33$$

- b. An iPod is purchased on installment basis, such that Rs 8000 is to be paid on the signing of the contract and four yearly installments of Rs 3000 each, payable at the end of the first, second, third and fourth years. If compound interest is charged at 5% per annum, what would be the cash price of the iPod ? [Take  $(1.05)^{-4} = 0.82271$ ].

**Answer:**

$$\text{Present worth} = \frac{A}{r} \left[ 1 - (1+r)^{-n} \right]$$

$$= \frac{3000}{.05} \left[ 1 - (1.05)^{-4} \right] = \frac{3000}{.05} \times (1 - .8227)$$

$$= \frac{300000}{5} (.1773)$$

$$= \text{Rs. } 10,638$$

$$\therefore \text{Cash price} = \text{Rs. } 8000 + 10638$$

$$= \text{Rs. } 18,638$$

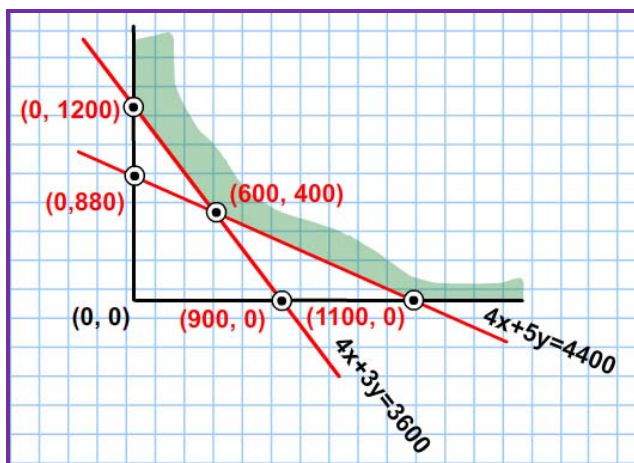
#### Question 14

[5+5=10]

- a. A new cereal, formed of a mixture of bran and rice, contains at least 88 grams of protein and at least 36 milligrams of iron. Knowing that bran contains 80 grams of protein and 40 milligrams of iron per kilogram, and that rice contains 100 grams of protein and 30 milligrams of iron per kilogram, find the minimum cost of producing a kilogram of this new cereal if bran costs Rs 28 per kilogram and rice costs Rs 25 per kilogram.

**Answer:**





Cost function =  $\frac{28x + 25y}{1000}$  where x gms brans and y gms rice.

$$8x + 10y \geq 8800$$

$$4x + 5y \geq 4400 \quad \dots\dots(i)$$

$$4x + 3y \geq 3600 \quad \dots\dots(ii)$$

$$x \geq 0, y \geq 0$$

$$x = 600 \text{ gm } y = 400 \text{ gm}$$

$$\text{minimum cost} = \frac{28 \times 600 + 25 \times 400}{1000}$$

$$= \frac{168 + 100}{10}$$

$$= \frac{268}{10}$$

$$= \text{Rs. } 26.8$$

- b. The cost of manufacturing of certain items consists of Rs 1600 as overheads, Rs 30 per 2 for x items produced. How item as the cost of the material and the labour cost Rs  $\frac{x^2}{100}$  many items must be produced to have a minimum-average cost ?

**Answer:**

$$\text{Cost function} = \frac{x^2}{100} + 30x + 1600$$

$$\text{Average} = \frac{x}{100} + 30 + \frac{1600}{x}$$



$$\frac{d}{dx}(\text{A.C.}) = \frac{1}{100} + 0 - \frac{1600}{x^2} = 0$$

$$\Rightarrow x = 400$$

$$\frac{d^2}{dx^2}(\text{A.C.}) = \frac{3200}{x^3} = \text{positive at } x = 4000$$

$\therefore$  A.C. is minimum at  $x = 4000$ .

$\therefore$  400 items must be produced

### Question 15

[3+5=8]

- a. Calculate the index number for the year 2006 with 1996 as the base year by the weighted average of price relatives method from the following data.

Commodity	A	B	C	D	E
Weight	40	25	20	5	10
Price (Rs per unit) Year 1996	32.00	80.00	1.00	10.20	4.00
Price (Rs per unit) Year 2006	40.00	120.00	1.00	15.36	3.00

**Answer:**

Commodity	Weight (W)	Price (Rs.) 1996	Price (Rs.) 2006	Price Relatives	W.x
A	40	32	40	$\frac{40}{32} \times 100 = 125$	5000
B	25	80	120	$\frac{120}{80} \times 100 = 150$	3750
C	5	1	1	$\frac{1}{1} \times 100 = 100$	500
D	20	10.24	15.36	$\frac{15.36}{10.24} \times 100 = 150$	3000
E	10	4.00	3.00	$\frac{3}{4} \times 100 = 75$	750
		$\Sigma W = 100$			$\Sigma Wx = 13000$

- a. A propeller costs Rs 180000 and its effective life is estimated to be 10 years. A sinking fund is created for replacing the propeller by a new model at the end of its life time, when its scrap realizes a sum of Rs 34000 only. The price of the new model is estimated to be 30% more than the price of the present one. What amount should be put into the sinking fund at the end of each year, if it accumulates at 4% per annum compound interest? [Take (1.04) power 10 = 1.480].

**Answer:**





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$$M = \frac{A}{r} \left[ (1+r)^n - 1 \right]$$

$$180000 + \frac{30}{100} \times 180000 - 34000 = \frac{A}{.04} [1.480 - 1]$$

$$200000 = \frac{A}{.04} \times .480 = 12A$$

$$\frac{200000}{12} = A$$

$$\therefore A = 16666.6$$

$$\therefore A = \text{Rs.}16667$$

*\*\* Out of syllabus. Answer should be provided up on request*

