
2015

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Section A (Question numbers 1 to 9)

Question: 1

[3x7=21]

- i. Given that $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ and $A (\text{adj. } A) = K \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find the value of K.

Answer:

$$A \text{ adj } A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix} \cdot A (\text{adj } A) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore k = 1$$

- ii. A straight line $y = mx$ passes through the intersection of the straight lines $x + 2y - 1 = 0$ and $2x - y + 3 = 0$. Find the value of m.

Answer:

Lines through the intersection are

$$x + 2y - 1 + k(2x - y + 3) = 0$$

As $y = mx$ pass through origin,

$$\therefore -1 + 3k = 0 \quad k = \frac{1}{3}$$

Thus the line is $x + 2y + \frac{2}{3}x - \frac{1}{3}y = 0$ or, $\frac{5}{3}x + \frac{5}{3}y = 0$ So $m = -1$ $m = -1$ or, $y = -x$

- iii. If $\sin(xy) + \cos(xy) = 1$ and $\tan(xy) \neq 1$, then show that $\frac{dy}{dx} = -\frac{y}{x}$.

Answer:

Let $z = xy$. So $\sin z + \cos z = 1$.

Differentiating w.r.to x, $(\cos z - \sin z) \frac{dz}{dx} = 0$

$$\therefore \frac{dz}{dx} = 0 \text{ as } \tan xy = \tan z \neq 1$$

$$\text{or, } y + x \frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = -\frac{y}{x}$$

- iv. Evaluate $\int \frac{\cos x}{\sin x + \sqrt{\sin x}} dx$.

Answer:

$$I = \int \frac{dz}{z + \sqrt{z}}, \quad z = \sin x$$

$$= \int \frac{1}{\sqrt{z}} \cdot \frac{dz}{1 + \sqrt{z}} \quad \sqrt{z} = t$$



$$\therefore \frac{dz}{\sqrt{z}} = 2dt = \int \frac{2dt}{1+t} = 2\log|1+t| + C = 2\log|1+\sqrt{\sin x}| + C$$

- v. Tickets numbered from 1 to 20 are mixed up together and then a ticket is drawn at random. What is the probability that the ticket has a number, which is multiple of 3 or 7.

Answer:

A ticket can be drawn from 20 tickets in 20 ways. Multiple of 3 or 7 occurs for the numbers 3, 6, 7, 9, 12, 14, 15, 18. Thus the event occurs in 8 ways.

$$\therefore \text{The probability} = \frac{8}{20} = \frac{2}{5}$$

- vi. A performance test was conducted among seven candidates. They scored the following marks: 7, 10, 12, 15, 17, 19, and 25. Calculate the interquartile range and the standard deviation.

Answer:

$$\text{Here } Q_1 = 10, Q_2 = 15, Q_3 = 19$$

$$\text{Interquartile range} = Q_3 - Q_1 = 19 - 10 = 9$$

$$\bar{x} = \frac{1}{n} \sum x_i = \frac{1}{7} (7 + 10 + 12 + 15 + 17 + 19 + 25) = \frac{105}{7} = 15$$

$$\text{Variance} = \frac{1}{n} \sum x_i^2 - \bar{x}^2 = \frac{1}{7} (49 + 100 + 144 + 225 + 289 + 361 + 625) - 15^2$$

$$\text{S.D.} = +\sqrt{\text{Variance}} \text{ etc.}$$

- vii. Find the amplitude of the complex number $\sin \frac{6\pi}{5} + i \left(1 - \cos \frac{6\pi}{5} \right)$.

Answer:

$$\sin \left(\frac{6\pi}{5} \right) + i.2\sin^2 \frac{3\pi}{5} = 2\sin \frac{3\pi}{5} \left(\cos \frac{3\pi}{5} + i.\sin \frac{3\pi}{5} \right). \text{ So amplitude is } \frac{3\pi}{5}$$

- viii. Solve the following differential equation $\frac{dy}{dx} - e^{y+x} = e^{x-y}$:

Answer:

$$\frac{dy}{e^y + e^{-y}} = e^x dx$$

$$\text{or, } \frac{e^y dy}{1 + e^{2y}} = e^x dx$$

$$\int \frac{dz}{1+z^2} = \int e^x dx \quad z = e^y$$

$$\text{or, } \tan^{-1}(e^y) = e^x + C \text{ etc.}$$



Question: 2

[5 × 2 = 10]

a. Using properties of determinants, show that

$$\begin{vmatrix} a^2 + 1 & ab & ac \\ ba & b^2 + 1 & bc \\ ca & cb & c^2 + 1 \end{vmatrix} = a^2 + b^2 + c^2 + 1$$

Answer:

The determinant = $\frac{1}{abc} \begin{vmatrix} a^3 + a & b^2a & c^2a \\ a^2b & b^3 + b & c^2b \\ a^2c & b^2c & c^3 + c \end{vmatrix}$ $c_1 \rightarrow ac_1, c_2 \rightarrow bc_2, c_3 \rightarrow cc_3$

$$= \begin{vmatrix} a^2 + 1 & b^2 & c^2 \\ a^2 & b^2 + 1 & c^2 \\ a^2 & b^2 & c^2 + 1 \end{vmatrix} \quad a, b, c \text{ are taken from } R_1, R_2 \text{ \& } R_3 \text{ respectively.}$$

$$= \begin{vmatrix} 1 + a^2 + b^2 + c^2 & b^2 & c^2 \\ 1 + a^2 + b^2 + c^2 & b^2 + 1 & c^2 \\ 1 + a^2 + b^2 + c^2 & b^2 & c^2 + 1 \end{vmatrix} \quad \text{By } c_1 + c_2 + c_3$$

$$= (1 + a^2 + b^2 + c^2) \begin{vmatrix} 1 & b^2 & c^2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad \text{Taking } 1 + a^2 + b^2 + c^2 \text{ from } c_1 \text{ \& } R_2 - R_1 \text{ \& } R_3 - R_1,$$

$$= 1 + a^2 + b^2 + c^2$$

b. Given the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ -2 & 1 & 0 \\ 0 & -1 & 2 \end{bmatrix}$ compute A^{-1} .

Answer:

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & -1 & 0 \\ 2 & 1 & 0 \end{vmatrix} = 2(1+2) = 6 \neq 0$$

$$A \text{ dj } A = \begin{pmatrix} 0 & +2 & 2 \\ 3 & -3 & 0 \\ 3 & 1 & -2 \end{pmatrix}. A^{-1} = \frac{1}{6} \begin{pmatrix} 0 & 2 & 2 \\ 3 & -3 & 0 \\ 3 & 1 & -2 \end{pmatrix}$$

$$\therefore X = \frac{1}{6} \begin{pmatrix} 0 & 2 & 2 \\ 3 & -3 & 0 \\ 3 & 1 & -2 \end{pmatrix} \begin{pmatrix} 6 \\ 2 \\ 1 \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 6 \\ 12 \\ 18 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

$$\therefore x = 1, y = 2, z = 3$$



Question: 3

[5]

The equation $2x^2 - 3xy - py^2 + x + qy = 0$ represents two perpendicular lines. Find the values of p and q .

Answer

Here, $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ gives

$$q \cdot \frac{1}{2} \cdot \left(-\frac{3}{2}\right) - 2 \cdot \left(\frac{q}{2}\right)^2 + p \cdot \left(\frac{1}{2}\right)^2 = 0$$

$$\text{or, } -\frac{3}{4}q - \frac{q^2}{2} + \frac{p}{4} = 0$$

$$\text{or, } -3q - 2q^2 + p = 0$$

For perpendicularity $2 - p = 0$ or, $p = 2$

So $2q^2 + 3q - 2 = 0$. Hence find q .

The bisectors are $\frac{x^2 - y^2}{2 - 3} = \frac{xy}{-\frac{7}{2}}$. The angle θ is $\tan \theta = \frac{2\sqrt{\frac{49}{4} - 6}}{2 + 3} = \frac{5}{5} = 1$

$$\theta = \frac{\pi}{4}$$

Question: 4

[5 × 2 = 10]

a. Prove that $\cot\left(\frac{\pi}{4} - 2\cot^{-1}3\right) = 7$

Answer:

We are to prove $\tan\left(\frac{\pi}{4} - 2\cot^{-1}3\right) = \frac{1}{7}$

$$\begin{aligned} \text{L.H.S} &= \frac{1 - \tan(2\cot^{-1}3)}{1 + \tan(2\cot^{-1}3)} = \frac{1 - \tan\left(2\tan^{-1}\frac{1}{3}\right)}{1 + \tan\left(2\tan^{-1}\frac{1}{3}\right)} = \frac{1 - \tan\left(\tan^{-1}\frac{\frac{2}{3}}{1 - \frac{1}{9}}\right)}{1 + \tan\left(\tan^{-1}\frac{\frac{2}{3}}{1 - \frac{1}{9}}\right)} \\ &= \frac{1 - \frac{6}{8}}{1 + \frac{6}{8}} = \frac{2}{14} = \frac{1}{7} \text{ Hence etc.} \end{aligned}$$

b. If $x^p y^q = (x + y)^{p+q}$. Prove that $\frac{dy}{dx} = \frac{y}{x}$

Answer:

Taking logarithm of both sides, $p \log x + q \log y = (p + q) \log(x + y)$



$$\text{Diff. w.r.to } x, \frac{p}{x} + \frac{q}{y} \cdot \frac{dy}{dx} = \frac{(p+q)}{x+y} \left(1 + \frac{dy}{dx} \right)$$

$$\text{or, } \left(\frac{q}{y} - \frac{p+q}{x+y} \right) \frac{dy}{dx} = \frac{p+q}{x+y} - \frac{p}{x}$$

$$\text{If } qx - py \neq 0 \text{ the result follows. or, } \frac{(qx - py)}{y} \cdot \frac{dy}{dx} = \frac{(qx - py)}{x} \cdot \frac{dy}{dx}$$

Question: 5

[5 × 2 = 10]

- a. Examine the validity and conclusion of Rolle's theorem for the function

$$f(x) = e^x \sin x, \forall x \in [0, \pi]$$

Answer:

$f(x)$ is continuous in $[0, \pi]$ and derivable in $(0, \pi)$. $f'(x) = e^x (\sin x + \cos x)$.

$$f(0) = 0 = f(\pi). f'(c) = 0 \text{ gives } \sin c = -\cos c$$

$$\text{or, } \tan c = -1 = \tan \left(\frac{\pi}{2} + \frac{\pi}{4} \right) = \tan \frac{3\pi}{4}$$

$$\therefore c = \frac{3\pi}{4}.$$

- b. Prove that the area of right-angled triangle of a given hypotenuse is maximum when the triangle is isosceles.

Answer:

$$\text{Area } A = \frac{1}{2} l^2 \sin \theta \cos \theta = \frac{1}{4} l^2 \sin 2\theta$$

$$\text{It is maximum when } \frac{dA}{d\theta} = 0$$

$$\cos 2\theta = 0$$

$$\theta = \frac{\pi}{4}$$

$$\left. \frac{d^2 A}{d\theta^2} \right|_{\theta = \frac{\pi}{4}} = l^2 (-\sin 2\theta) \Big|_{\theta = \frac{\pi}{4}} = -l^2 < 0$$

Question: 6

[5 × 2 = 10]

- a. Prove that $\int_0^{\frac{\pi}{2}} \frac{3 \sin \theta + 4 \cos \theta}{\sin \theta + \cos \theta} d\theta = \frac{7\pi}{4}$.

Answer:

$$I = \int_0^{\frac{\pi}{2}} \frac{3 \sin \theta + 4 \cos \theta}{\sin \theta + \cos \theta} d\theta = \int_0^{\frac{\pi}{2}} \frac{3 \sin \theta + 3 \cos \theta}{\sin \theta + \cos \theta} d\theta$$

$$\therefore 2I = 7 \cdot \frac{\pi}{2}$$



$$\therefore I = \frac{7p}{4}$$

- b. Calculate the area bounded by the curve $y = x(2 - x)$ and the lines $x = 0$, $y = 0$, $x = 2$. This area is rotated through four right angles about the x - axis. Calculate the volume of the solid so formed.

Answer:

$$\text{The area} = \int_0^2 y \, dx = \int_0^2 (2x - x^2) \, dx = \left(x^2 - \frac{1}{3} x^3 \right) \Big|_0^2 = \left(2^2 - \frac{2^3}{3} \right) \text{ etc.}$$

$$\text{The volume} = \frac{\pi}{4} \int_0^2 y^2 \, dx = \frac{\pi}{4} \int_0^2 (2x - x^2)^2 \, dx \text{ etc.}$$

Question: 7

[5 × 2 = 10]

- a. Internal and External assessments were conducted on a group of 10 students who were studying in a postgraduate class in a college. The following marks were obtained in the assessments:

Roll. No. of students	1	2	3	4	5	6	7	8	9	10
Internal Assessment	45	62	67	32	12	38	47	68	42	85
External Assessment	39	48	65	32	20	35	45	77	30	62

Find the Spearman's Rank Correlation Coefficient and comment on the result.

(**)

Answer:

- b. There are two series of index numbers: P for price index and S for stock of a commodity. The mean and standard deviation of P are 100 and 8 and so S are 103 and 4 respectively. The correlation coefficient between the series is 0.4. With these data, obtain the regression lines of P on S and S on P.

Answer:

Let mean of P series is \bar{y} , its s.d. is σ_y and those of S series are respectively \bar{x} and σ_x .

$$\therefore \bar{y} = 100, \quad \sigma_y = 8, \quad \bar{x} = 103 \text{ and } \sigma_x = 4 \quad r = .4$$

$$\text{So regression line of P on S is } y - \bar{y} = r \cdot \frac{\sigma_y}{\sigma_x} (x - \bar{x})$$

$$\text{i.e, } y - 100 = 4 \cdot \frac{8}{4} (x - 103)$$

$$\text{Similarly regression line of S on P is } x - \bar{x} = r \cdot \frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

$$\text{i.e, } x - 103 = 4 \cdot \frac{4}{8} (y - 100) \text{ etc.}$$



Question: 8

[5 × 2 = 10]

- a. In a certain city, the probability of not reading the morning newspaper by the residents is $\frac{1}{2}$ and that of not reading the evening newspaper is $\frac{2}{5}$. The probability of reading both the newspaper is $\frac{1}{5}$. Find the probability that a resident reads either the morning or evening or both the papers.

Answer:

Let A: reading the morning newspaper, B: reading the evening newspaper

The probability is $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{3}{5} - \frac{1}{5}$ etc.

- b. A candidate is selected for interview of management trainees for 3 companies. For the first company, there are 12 candidates, for the second there are 15 candidates and for the third, there are 10 candidates. Find the probability that he is selected in at least one of the companies.

Answer:

A: To be selected in first company $P(A) = \frac{1}{12}$

B: To be selected in second company $P(B) = \frac{1}{15}$

C: To be selected in third company $P(C) = \frac{1}{10}$

A, B, C are independent events and so are $\bar{A}, \bar{B}, \bar{C}$

$$P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A})P(\bar{B})P(\bar{C}) = \frac{11}{12} \cdot \frac{14}{15} \cdot \frac{9}{10}$$

$$\therefore P(A \cup B \cup C) = 1 - P(\bar{A} \cap \bar{B} \cap \bar{C}) = 1 - \frac{11}{12} \cdot \frac{14}{15} \cdot \frac{9}{10}$$

Question: 9

[5 × 2 = 10]

- a. Using De Moivre's theorem, find the least value of $n \in \mathbb{N}$ for which the expression

$$(1+i)^n (1-i)^n \text{ is equal to } -2^{\frac{n+2}{2}}.$$

Answer:

$$(1+i)^n = (\sqrt{2})^n \left(\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right)^n = (\sqrt{2})^n \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)^n = (\sqrt{2})^n \cdot \left(\cos \frac{n\pi}{4} + i \sin \frac{n\pi}{4} \right)^n$$

$$(1-i)^n = (\sqrt{2})^n \cdot \left(\cos \frac{n\pi}{4} - i \sin \frac{n\pi}{4} \right) = (1+i)^n + (1-i)^n = (\sqrt{2})^n \cdot 2 \cos \frac{n\pi}{4} = 2^{\frac{n+2}{2}} \cos \frac{n\pi}{4}$$

\therefore least value of n is 4.



b. Solve the differential equation $\tan x \frac{dy}{dx} + 2y = \sec x$

Answer:

$\frac{dy}{dx} + 2\cot x \cdot y = \operatorname{cosec} x$. This is linear in y .

$$\text{I.F} = e^{\int 2\cot x dx} = e^{\log \sin^2 x} = \sin^2 x$$

\therefore the solution is $y \sin^2 x = -\cos x + C$



Section B (Question numbers 10 to 12)

Question: 10

[5 × 2 = 10]

- a. A plane passes through the point (4, 2, 4) and is perpendicular to the planes $2x + 5y + 4z + 1 = 0$ and $4x + 7y + 6z + 2 = 0$. Find the equation of the plane.

Answer:

The planes through (1, 2, 3) is: $a(x-1) + b(y-2) + c(z-3) = 0$

From perpendicularity condition $a + b + 2c = 0$, $3a + 2b + c = 0$

$$\therefore \frac{a}{-3} = \frac{b}{5} = \frac{c}{-1}$$

So the plane is $3(x-1) - 5(y-2) + (z-3) = 0$.

- b. Find the least distance of the plane $12x + 4y + 3z = 327$ from the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$.

Answer:

Centre of the sphere is $C(-2, 1, 3)$. Its radius = $\sqrt{4+1+9+155} = 13$. Perpendicular distance of C

$$\text{from the plane} = \left| \frac{12(-2) + 4(1) + 3(3) - 327}{\sqrt{144 + 16 + 9}} \right| = \frac{338}{13} = 26$$

So the least distance = $26 - 13 = 13$

Question: 11

[5 × 2 = 10]

- a. \vec{a} and \vec{b} are unit vectors such that $2\vec{a} - 4\vec{b}$ and $10\vec{a} + 8\vec{b}$ are perpendicular to each other. Find the angle between the vectors \vec{a} and \vec{b} .

Answer:

$$(2\vec{a} - 4\vec{b}) \cdot (10\vec{a} + 8\vec{b}) = 0$$

$$\text{or, } 20 - 24\vec{a} \cdot \vec{b} - 32 = 0 \quad \text{or, } \vec{a} \cdot \vec{b} = -\frac{1}{2} \quad \therefore \cos \theta = -\frac{1}{2}$$

$$\theta = 120^\circ$$

- b. Prove that $\vec{a}(\vec{b} + \vec{c}) \times (\vec{a} + 2\vec{b} + 3\vec{c}) = [\vec{a} \vec{b} \vec{c}]$.

Answer:

$$\begin{aligned} & \vec{a} \{ (\vec{b} + \vec{c}) \times (\vec{a} + 2\vec{b} + 3\vec{c}) \} \\ &= \vec{a} \cdot \{ \vec{b} \times \vec{a} + 3\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + 2\vec{c} \times \vec{b} \} \\ &= 3\vec{a}(\vec{b} \times \vec{c}) + 2\vec{a}(\vec{c} \times \vec{b}) \\ &= [\vec{a} \vec{b} \vec{c}] \end{aligned}$$



Question: 12

[5 × 2 = 10]

- a. Calculate the index number for the year 1979 with 1970 as base from the following data using weighted average of price relatives:

Answer:

Price relatives are $\frac{180}{140} \times 100$, $\frac{550}{400} \times 100$, $\frac{250}{100} \times 100$, $\frac{150}{120} \times 100$ and $\frac{300}{200} \times 100$

i.e., 128.57, 137.5, 250, 125, 150

$$\therefore \text{Price index} = \frac{128.57 \times 10 + 137.5 \times 7 + 250 \times 6 + 125 \times 8 + 150 \times 4}{10 + 7 + 6 + 8 + 4}$$

$$= \frac{1285.7 + 962.5 + 1500 + 1000 + 600}{35} \text{ etc.}$$

Commodity	Weight	Price in Rs.	
		1970	1979
A	22	2.50	6.20
B	48	3.30	4.40
C	17	6.25	12.75
D	13	0.65	0.90

- b. The average number, in lakhs, of working days lost in strikes during each year of the period 1981 – 90 was:

1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
1.5	1.8	1.9	2.2	2.6	3.7	2.2	6.4	3.6	5.4

Calculate the three – yearly moving averages and draw the moving averages graph.

Answer:

Year	Working days last	3 yr moving total	3 yr moving averages
1981	1.5	-	-
1982	1.8	5.2	1.73
1983	1.9	5.9	1.97
1984	2.2	6.7	2.23
1985	2.6	8.5	2.83
1986	3.7	8.5	2.83
1987	2.2	12.3	4.10
1988	6.4	12.2	4.07
1989	3.6	15.4	5.13
1990	5.4	-	-

etc.



Section C (Question numbers 13 to 15)**Question 13**

[5 × 2 = 10]

- a. A bill of exchange for Rs.750.00 was drawn on 3rd April, 2000 payable at 3 months after date. It was discounted on 24th April, 2000 at 5% per annum. What was the discounted value of the bill?

Answer:

Price relatives are $\frac{180}{140} \times 100, \frac{550}{400} \times 100, \frac{250}{100} \times 100, \frac{150}{120} \times 100$ and $\frac{300}{200} \times 100$

i.e., 128.57, 137.5, 250, 125, 150

$$\begin{aligned}\therefore \text{Price index} &= \frac{128.57 \times 10 + 137.5 \times 7 + 250 \times 6 + 125 \times 8 + 150 \times 4}{10 + 7 + 6 + 8 + 4} \\ &= \frac{1285.7 + 962.5 + 1500 + 1000 + 600}{35} \text{ etc.}\end{aligned}$$

Commodity	Price in Rs. in year 2000	Price in Rs. In year 2005	Weight
A	140	180	10
B	400	550	7
C	100	250	6
D	125	150	8
E	200	300	4

- b. Find the amount of an ordinary annuity if payment of Rs.600.00 is made at the end of every quarter for 10 years at the rate of 4% per year compounded quarterly?

Answer:

$$\text{Amount of ordinary annuity} = M = \frac{A}{r} \left[(1+r)^n - 1 \right]$$

$$\text{Here } A = \text{Rs.}600.00, n = 40, r = \frac{4}{100 \times 4} = .01 \quad r = \frac{4}{100 \times 4} = 0.1 \text{ compounded quarterly}$$

$$\therefore M = \text{Rs.} \frac{600}{.01} \left[(1.01)^{40} - 1 \right] \text{ etc.}$$

Question 14

[5 × 2 = 10]

- a. How much should a company set aside at the end of each year if it has to buy a machine expected to cost Rs.100,000 at the end of 4 years and the rate of interest is 5% per annum compounded annually?

Answer:

$$\text{From } M = \frac{A}{r} \left[(1+r)^n - 1 \right] \text{ we have } 1,00,000 = \frac{A}{.05} \left[(1.05)^4 - 1 \right]$$

- b. A company is selling a certain product. The demand function of the product is linear. The company can sell 2000 units when the price is Rs.8 per unit and when the price is Rs.4 per unit, it can sell 3000 units. Determine:

- i. The demand function



ii. The total revenue function

[5]

Answer:

Demand function $p = a + bx$. p is the price per unit, x is the number of units, a and b are constants.

$$\therefore 8 = a + 2000b$$

$$4 = a + 3000b$$

$$\text{So } 4 = -1000b \text{ and } a = 8 + 8 = 16$$

$$\therefore p = 16 - \frac{1}{250}x \text{ is the demand function.}$$

$$\text{Total revenue } R = px = 16x - \frac{1}{250}x^2$$

Question 15

[5 × 2 = 10]

- a. A manufacturing firm produces steel pipes in three plants A, B and C with daily production of 500, 1000 and 2000 units respectively. The fractions of defective steel pipes output produced by the plants A, B and C are respectively .005, .008 and .010. If a pipe is selected from a day's total production and found to be defective, find out the probability that it came from the first plant.

Answer:

$$\text{Let } A_1: \text{ Pipe produced in plant A; } P(A_1) = \frac{5}{35} = \frac{1}{7}$$

$$A_2: \text{ Pipe produced in plant B; } P(A_2) = \frac{10}{35} = \frac{2}{7}$$

$$A_3: \text{ Pipe produced in plant C; } P(A_3) = \frac{20}{35} = \frac{4}{7}$$

X: Defective pipe.

$$\therefore P\left(\frac{X}{A_1}\right) = .005, P\left(\frac{X}{A_2}\right) = .008, P\left(\frac{X}{A_3}\right) = .01$$

$$\text{By Baye's theorem, } P\left(\frac{A_1}{X}\right) = \frac{P(A_1)P\left(\frac{X}{A_1}\right)}{P(A_1)P\left(\frac{X}{A_1}\right) + P(A_2)P\left(\frac{X}{A_2}\right) + P(A_3)P\left(\frac{X}{A_3}\right)}$$

- b. A coin is tossed 5 times. What is the probability of getting at least three heads?

Answer:

$$\text{Probability of 'success' i.e., 'head' } = \frac{1}{2} = p$$

$$\text{So probability of } i \text{ 'heads' } = {}^5C_i \left(\frac{1}{2}\right)^i \left(\frac{1}{2}\right)^{5-i} = {}^5C_i \cdot \frac{1}{2^5}$$

$$\therefore \text{The probability of at least three heads} = \sum_{i=3}^5 {}^5C_i \cdot \frac{1}{2^5} \text{ etc.}$$

**** Out of syllabus. Answer should be provided up on request**

