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**2015**

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**Question: 1** (Answer any one questions)

[2x1=2]

a.

i. Find the differential equation of the family of lines passing through the origin.

**Answer:**

Equation of line passing through the origin is:

$$y - 0 = m(x - 0), \text{ or}$$

$$\Rightarrow y = mx, \text{ or}$$

$$\frac{dy}{dx} = m \quad (1)$$

Now, substituting,  $m = \frac{dy}{dx}$  in equation (1) as,

$$y = mx, \text{ or}$$

$$y = \left(\frac{dy}{dx}\right)x$$

Therefore,  $y = \left(\frac{dy}{dx}\right)x$  is the required differential equation.

ii. If  $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ , and  $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$ . Then, find  $|\vec{a} \times \vec{b}|$ .

**Answer:**

From given value we get

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix}$$

$$= \hat{i}\{1 \cdot (-2) - 5 \cdot 3\} - \hat{j}\{2 \cdot (-2) - 3 \cdot 3\} + \hat{k}\{2 \cdot 5 - 3 \cdot 1\}$$

$$= \hat{i}\{-2 - 15\} - \hat{j}\{-4 - 9\} + \hat{k}\{10 - 3\}$$

$$= \hat{i}(-17) - \hat{j}(-13) + \hat{k}(7)$$

$$= 17\hat{i} + 13\hat{j} + 7\hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(17)^2 + (13)^2 + (7)^2} = \sqrt{289 + 169 + 49} = \sqrt{507}$$

b. Answer any one question:

Evaluate:

$$\text{i. } \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [2 \ 3 \ 4] \text{ and}$$

**Answer:**

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [2 \ 3 \ 4] = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 4 \end{bmatrix}$$



ii.  $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

**Answer:**

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1-4 & 2-6 & 3-2 \\ 2+6 & 4+9 & 6+3 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

c. Answer any three questions:

i. Find the principal value of  $\sin^{-1} \left( \frac{1}{\sqrt{2}} \right)$ .

**Answer:**

Lets  $\sin^{-1} \left( \frac{1}{\sqrt{2}} \right) = y$

$$\therefore \sin y = \frac{1}{\sqrt{2}}$$

$$= \sin \left( \frac{\pi}{4} \right)$$

$$\therefore \sin y = \frac{\pi}{2}$$

Hence, principal value of  $\sin^{-1} \left( \frac{1}{\sqrt{2}} \right) = \frac{\pi}{4}$

ii. Evaluate:  $\int \cos^4 x dx$ .

**Answer:**

$$\int \cos^4 x dx = \int (\cos^2 x)^2 dx$$

$$= \int \left( \frac{1 + \cos 2x}{2} \right)^2 dx$$

$$= \frac{1}{4} \int (1 + 2\cos 2x + \cos^2 2x) dx$$

$$= \frac{1}{4} \int \left( 1 + 2\cos 2x + \frac{1 + \cos 4x}{2} \right) dx$$

$$= \frac{1}{4} \int \left( \frac{3}{2} + 2\cos 2x + \frac{1}{2} \cos 4x \right) dx$$

$$= \frac{1}{4} \left[ \frac{3}{2}x + 2 \cdot \frac{\sin 2x}{2} + \frac{1}{2} \cdot \frac{\sin 4x}{4} \right] + c$$

$$= \frac{3}{8}x + \frac{\sin 2x}{2} + \frac{\sin 4x}{32} + c$$

iii. Using differentials, find the approximate value of  $\sqrt{26}$ .



**Answer:**

$$\text{Let } f(x) = \sqrt{x} \Leftrightarrow f'(x) = \frac{1}{2\sqrt{x}}.$$

$$\text{For } \sqrt{26} = \sqrt{25+1},$$

take  $x_0 = 25$

and  $h = 1$ .

$$\therefore f(x_0) = f(25) = \sqrt{25} = 5;$$

$$f'(x_0) = \frac{1}{2 \times 5}$$

$$= 0.1$$

Using the formula for approximation,

$f(x_0 + h) = f(x_0) + hf'(x_0)$ , we have

$$\Leftrightarrow \sqrt{26} = 5 + 1 \times 0.1 = 5.1.$$

iv. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = 3x - 2$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $g(x) = \frac{x+2}{3}$  show that  $f \circ g = I_{\mathbb{R}}$ .

**Answer:**

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{x+2}{3}\right) = 3\left(\frac{x+2}{3}\right) - 2$$

$$= x + 2 - 2$$

$$= x = I_{\mathbb{R}}(x)$$

$$\therefore (f \circ g) = I_{\mathbb{R}}$$

v. Using the property of determinants, = 0

**Answer:**

Applying  $C_1 \rightarrow C_2 + C_3$

$$\begin{vmatrix} a-b+b-c+c-a & b-c & c-a \\ b-c+c-a+a-b & c-a & a-b \\ c-a+a-b+b-c & a-b & b-c \end{vmatrix} = \begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 0 & a-b & b-c \end{vmatrix}$$

$$= 0 \left[ \text{Since the elements of } C_1 \text{ are all zero} \right]$$

$$\text{vi. Evaluate : } \sin \left[ \frac{\pi}{3} - \sin^{-1} \left( -\frac{1}{2} \right) \right]$$

**Answer:**

$$\sin \left[ \frac{\pi}{3} - \sin^{-1} \left( -\frac{1}{2} \right) \right]$$

$$= \sin \left[ \frac{\pi}{3} + \sin^{-1} \frac{1}{2} \right]$$



$$= \sin \left[ \frac{\pi}{3} + \frac{\pi}{6} \right]$$

$$= \sin \frac{\pi}{2}$$

$$= 1$$

d. Answer any one question:

i. For what value of k is the following function continuous at x=2?  $f(x) = \begin{cases} 2x+1, & x \neq 2 \\ k, & x = 2 \\ 3x-1, & x \neq 2 \end{cases}$

**Answer:**

$\therefore f(x)$  is continuous at  $x = 2$

$$\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} f(x) = f(2)$$

$$\lim_{x \rightarrow 2} 2x+1 = \lim_{x \rightarrow 2} 3x-1 = k$$

$$\text{consider, } \lim_{x \rightarrow 2} 3x-1 = k$$

$$6-1 = k \Rightarrow k = 5$$

ii. Evaluate:  $\int \sec^2(7-4x) dx$

**Answer:**

$$\text{Let } I = \int \sec^2(7-4x) dx$$

$$\text{Let } 7-4x = m, -4dx = dm$$

$$\Rightarrow I = \frac{-1}{4} \tan m + c = -\frac{1}{4} \tan(7-4x) + c$$

e. Answer any one question:

i. Prove the following:  $\tan^{-1} x + \tan^{-1} \left( \frac{2x}{1-x^2} \right) = \tan^{-1} \left( \frac{3x-x^3}{1-3x^2} \right)$

**Answer:**

$$\text{LHS} = \tan^{-1} \tan^{-1} \left[ \frac{x + \frac{2x}{1-x^2}}{1 - x \frac{2x}{1-x^2}} \right]$$

$$= \tan^{-1} \left[ \frac{x(1-x^2) + 2x}{1-x^2 - 2x^2} \right]$$

$$= \tan^{-1} \left[ \frac{3x-x^3}{1-3x^2} \right] = \text{RHS}$$

ii. Prove the following:  $\cos[\tan^{-1}\{\sin(\cot^{-1} x)\}] = \sqrt{\frac{1+x^2}{2+x^2}}$



**Answer:**

$$\begin{aligned}\text{LHS} &= \cos[\tan^{-1}\{\sin(\cot^{-1} x)\}] \\&= \cos\left[\tan^{-1}\left\{\sin\left(\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)\right\}\right] \\&= \cos\left[\tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right] \\&= \cos\left[\cos^{-1}\frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}\right] \\&= \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}} \text{ R.H.S}\end{aligned}$$

**Question: 2**

a. Answer any one question:

i. Evaluate  $\int (1-x)\sqrt{x} \, dx$ .

**Answer:**

$$\begin{aligned}I &= \int (1-x)\sqrt{x} \, dx \\ \Rightarrow I &= \int (\sqrt{x} - x) \, dx \\ \Rightarrow I &= \int x^{\frac{1}{2}} \, dx - \int x^{\frac{3}{2}} \, dx \\ \Rightarrow I &= \frac{x^{\left(\frac{3}{2}\right)}}{\left(\frac{3}{2}\right)} - \frac{x^{\left(\frac{5}{2}\right)}}{\left(\frac{5}{2}\right)} + C \\ \Rightarrow I &= \frac{2}{3} x^{\left(\frac{3}{2}\right)} - \frac{2}{5} x^{\left(\frac{5}{2}\right)} + C\end{aligned}$$

ii. If  $\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} \begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} -4 & 6 \\ -9 & x \end{pmatrix}$ , write the value of x.

**Answer:**

$$\begin{aligned}\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} \begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix} &= \begin{pmatrix} -4 & 6 \\ -9 & x \end{pmatrix} \\ \Rightarrow \begin{pmatrix} 2 & -6 & -6 & +12 \\ 5 & -14 & -15 & +28 \end{pmatrix} &= \begin{pmatrix} -4 & 6 \\ -9 & x \end{pmatrix}\end{aligned}$$



$$\Rightarrow \begin{pmatrix} -4 & 6 \\ -9 & 13 \end{pmatrix} = \begin{pmatrix} -4 & 6 \\ -9 & x \end{pmatrix}$$

By the equality of matrices, we have  $x = 13$ .

b. Answer the following questions:

[3]

Show that  $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$ , where  $a, b, c$  are in A.P.

**Answer:**

$\therefore a, b, c$  are in A.P.

$$\therefore b - a = c - b \quad (1)$$

$$\text{Now L.H.S.} = \begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

$$\text{Operate: } R_2 \rightarrow R_2 - R_1; R_3 \rightarrow R_3 - R_2 = \begin{vmatrix} x+1 & x+2 & x+a \\ 1 & 1 & b-a \\ 1 & 1 & c-a \end{vmatrix} = \begin{vmatrix} x+1 & x+2 & x+3 \\ 1 & 1 & b-a \\ 1 & 1 & c-a \end{vmatrix} \quad [\text{Using (1)}]$$

$$= 0 \quad [\because R_2 \text{ and } R_3 \text{ are identical}] = \text{R.H.S.}$$

OR

Find the co-ordinates of the foot of the perpendicular drawn from the point  $A(1, 8, 4)$  to the line joining the points  $B(0, -1, 3)$  and  $C(2, -3, -1)$ .

**Answer:**

$$\text{Equation of line through } B(0, -1, 3) \text{ and } C(2, -3, -1) \text{ is } \frac{x-0}{0-2} = \frac{y+1}{-1-3} = \frac{z-3}{3+1} \Rightarrow \frac{x}{-2} = \frac{y+1}{-4} = \frac{z-3}{4}$$

Any point on (1) is  $P(-r, r-1, 2r+3)$ .

Let now  $AP \perp BC$

d. r. s. of  $BC$  are  $-1, 1, 2$

d. r. s. of  $AP$  are  $-r-1, r-1-8, 2r+3-4$

i.e.,  $-(r+1), r-9, 2r-1$

$$\therefore (r+1).1 + (r-9).1 + (2r-1).2 = 0 \Rightarrow 6r - 10 = 0 \Rightarrow r = \frac{5}{3}$$

$$\therefore \Rightarrow \left( -\frac{5}{3}, -\frac{5}{3} - 1, \frac{10}{3} + 3 \right) = \left( -\frac{5}{3}, -\frac{8}{3}, \frac{19}{3} \right) \text{ This is the required foot of the perpendicular,}$$

c. Answer the following questions:

Express the vector  $\vec{a} = 5\hat{i} - 2\hat{j} + 5\hat{k}$  as sum of two vectors such that one is parallel to the vector  $\vec{b} = 3\hat{i} + \hat{k}$  and the other is perpendicular to  $\vec{b}$ .



**Answer:**

Here  $\vec{a} = 5\hat{i} - 2\hat{j} + 5\hat{k}$

Any vector parallel to  $\vec{b} = 3\hat{i} + \hat{k}$  is  $\lambda \vec{b}$

Let  $\vec{c}$  be the vector  $\perp$  to  $\vec{b}$  s.t.

$$\vec{a} = \vec{c} + \lambda \vec{b}$$

$$\Rightarrow \vec{c} = \vec{a} - \lambda \vec{b}$$

$$\text{Now } \vec{b} \cdot \vec{c} = 0 \Rightarrow \vec{b} \cdot (\vec{a} - \lambda \vec{b}) = 0 \Rightarrow \vec{b} \cdot \vec{a} - \lambda \vec{b} \cdot \vec{b} = 0 \Rightarrow (3\hat{i} + \hat{k}) \cdot (5\hat{i} - 2\hat{j} + 5\hat{k}) - (3\hat{i} + \hat{k}) \cdot (3\hat{i} + \hat{k}) = 0$$

$$\Rightarrow 15 + 5 - \lambda(9 + 1) = 0 \Rightarrow \lambda = 2$$

$\therefore$  Vector parallel to  $\vec{b}$  is  $2\vec{b} = 6\hat{i} + 2\hat{k}$

$$\text{Vector } \perp \text{ to } \vec{b} \text{ is } \vec{c} = \vec{a} - 2\vec{b} = (5\hat{i} - 2\hat{j} + 5\hat{k}) - (6\hat{i} + 2\hat{k}) = -\hat{i} - 2\hat{j} + 3\hat{k}$$

OR

Find  $\lambda$  so that the four points with position vectors  $-6\hat{i} + 3\hat{j} + 2\hat{k}$ ,  $3\hat{i} + \lambda\hat{j} + 4\hat{k}$ , and  $5\hat{i} + 7\hat{j} + 3\hat{k}$  and  $-13\hat{i} + 17\hat{j} - \hat{k}$  are coplanar.

**Answer:**

Let  $A(-6\hat{i} + 3\hat{j} + 2\hat{k})$ ;  $B(3\hat{i} + \lambda\hat{j} + 4\hat{k})$ ;

$C(5\hat{i} + 7\hat{j} + 3\hat{k})$ ;  $D(-13\hat{i} + 17\hat{j} - \hat{k})$

$$\Leftrightarrow \vec{AB} = (3\hat{i} + \lambda\hat{j} + 4\hat{k}) - (-6\hat{i} + 3\hat{j} + 2\hat{k})$$

$$= 9\hat{i} + (\lambda - 3)\hat{j} + 2\hat{k}$$

$$\vec{AC} = 11\hat{i} + 4\hat{j} + \hat{k}$$

$$\vec{AD} = 7\hat{i} + 14\hat{j} - 3\hat{k}$$

Now A, B, C, D are coplanar.  $[\vec{AB} \ \vec{AC} \ \vec{AD}] = 0$

$$\Leftrightarrow \begin{vmatrix} 9 & \lambda - 3 & 2 \\ 11 & 4 & 1 \\ -7 & 14 & -3 \end{vmatrix} = 0 \Leftrightarrow 9(-12 - 14) - (\lambda - 3)(-33 + 7) + 2(154 + 28) \Leftrightarrow \lambda = -2$$

OR

By hypothesis, perimeter of the window = 20

Using properties of integral, evaluate :  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x) dx$ , where  $f(x) = \sin|x| + \cos|x|$

**Answer:**

$$\text{Let } I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x) dx$$

$$\text{where } f(x) = \sin|x| + \cos|x| \Leftrightarrow f(-x) = \sin|-x| + \cos|-x| = \sin|x| + \cos|x| = f(x)$$





$$\begin{aligned}
 \therefore I &= 2 \int_0^{\frac{\pi}{2}} f(x) dx \quad [Q \ f(x) \text{ is even function}] \\
 &= 2 \int_0^{\frac{\pi}{2}} [\sin|x| + \cos|x|] dx = 2 \int_0^{\frac{\pi}{2}} [\sin x + \cos x] dx \quad [Q|x| = x \text{ in } < x < \frac{\pi}{2}] = 2 [-\cos x + \sin x]_0^{\frac{\pi}{2}} \\
 &= 2 \left[ -\left(\cos \frac{\pi}{2} - \cos 0\right) + \left(\sin \frac{\pi}{2} - \sin 0\right) \right] = 4.
 \end{aligned}$$

OR

Ramesh appears for an interview for two posts A and B for which selection is independent. The probability of his selection for post A is  $\frac{1}{6}$  and for post B is  $\frac{1}{7}$ . Find the probability that Ramesh is selected for at least one of the posts.

**Answer:**

$$\text{Given : } P(A) = \frac{1}{6}; P(B) = \frac{1}{7}$$

Since selection for posts A and b is independent  $\Rightarrow P(A \cap B) = P(A) \cdot P(B)$

$$\begin{aligned}
 &= \frac{1}{6} \times \frac{1}{7} \\
 &= \frac{1}{42}.
 \end{aligned}$$

Now  $P(\text{selection for at least one post}) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\begin{aligned}
 &= \frac{1}{6} + \frac{1}{7} - \frac{1}{42} \\
 &= \frac{12}{42} \\
 &= \frac{2}{7}.
 \end{aligned}$$

OR

$$\text{Prove that } \tan^{-1}x + \tan^{-1} \frac{2x}{1-x^2} = \tan^{-1} \left( \frac{3x - x^3}{1-3x^2} \right)$$

**Answer:**

$$\begin{aligned}
 \text{L.H.S} &= \tan^{-1}x + \tan^{-1} \frac{2x}{1-x^2} \\
 &= \tan^{-1} \left[ \frac{x + \frac{2x}{1-x^2}}{1 - x \cdot \frac{2x}{1-x^2}} \right]
 \end{aligned}$$



$$\begin{aligned}
&= \tan^{-1} \left[ \frac{\frac{x(1-x^2) + 2x}{1-x^2}}{\frac{1-x^2-2x^2}{1-x^2}} \right] \\
&= \tan^{-1} \left[ \frac{x-x^3+2x}{1-x^2} \cdot \frac{1-x^2}{1-3x^2} \right] \\
&= \tan^{-1} \left( \frac{3x-x^3}{1-3x^2} \right)
\end{aligned}$$

**Question: 3**

a. Answer any one question:

[5x1=5]

i. Find the derivative of  $\tan x^2$  w.r.t.  $x$  from the first principle.

**Answer:**

Let  $y = \tan x^2$

Let  $\delta x$  be a small increment in  $x$  and  $\delta y$  be the corresponding increment in the value of  $y$ .

$$\therefore y + \delta y = \tan(x + \delta x)^2$$

Now (2) - (1)  $\Rightarrow$

$$\begin{aligned}
\delta y &= \tan(x + \delta x)^2 - \tan x^2 \\
&= \frac{\sin(x + \delta x)^2}{\cos(x + \delta x)^2} - \frac{\sin x^2}{\cos x^2} \\
&= \frac{\sin(x + \delta x)^2 \cos x^2 - \cos(x + \delta x)^2 \sin x^2}{\cos(x + \delta x)^2 \cos x^2} \\
&= \frac{\sin[(x + \delta x)^2 - x^2]}{\cos(x + \delta x)^2 \cos x^2} \\
\Rightarrow \frac{\delta y}{\delta x} &= \frac{\sin[(2x + \delta x)\delta x]}{\cos(x + \delta x)^2 \cos x^2} \cdot \frac{1}{\delta x} \\
&= \frac{\sin[(2x + \delta x)\delta x]}{[(2x + \delta x)\delta x]} \cdot \frac{2x + \delta x}{\cos(x + \delta x)^2 \cos x^2}
\end{aligned}$$

Now proceed to limits as  $\delta x \rightarrow 0$ .

$$\therefore \frac{dy}{dx} = 1 \cdot \frac{2x}{\cos x^2 \cos x^2} = 2x \sec^2 x^2$$



ii. Evaluate :  $\int \frac{\sin(2\tan^{-1}x)}{1+x^2} dx$ .

**Answer:**

$$\text{Let } I = \int \frac{\sin(2\tan^{-1}x)}{1+x^2} dx$$

$$\text{put : } \tan^{-1}x = t \Rightarrow \frac{dx}{1+x^2} = dt$$

$$\therefore I = \int \sin 2t \, dt = \frac{1}{2} \cos 2t + c$$

$$= \frac{1}{2} \cos(2 \tan^{-1}x) + c.$$

b. Answer any two questions:

[5x2=10]

i. Find the value of  $\int_0^2 (2x^2 - 3) dx$  as limit of sums.

**Answer:**

Here  $f(x) = 2x^2 - 3$ ;

$A = 0$ ,  $b = 2$  and  $nh = 2 - 0 = 2$

$$\therefore \int_0^2 (2x^2 - 3) dx$$

$$= \lim_{h \rightarrow 0} \left[ f(0) + f(0) + f(0+2h) + \dots + f(0 + (n-1)h) \right]$$

$$= \lim_{h \rightarrow 0} \left[ -3 + (2h^2 - 3) + \{2(2h^2) - 3\} + \dots + (n-1)h^2 \right]$$

$$= \lim_{h \rightarrow 0} \left[ -3nh + \frac{1}{3} + 2h^3 \frac{(n-1)n(2n-1)}{6} \right]$$

$$= \lim_{h \rightarrow 0} \left[ -3 \times 2 + \frac{1}{3} (2-0) \times 2 (2 \times 2 - 0) \right]$$

( $\because nh = 2$  and  $h \rightarrow 0$ )

$$= -6 + \frac{16}{3} = -\frac{2}{3}$$

ii. Draw a rough sketch and find the area of the region bounded by the two parabolas  $y^2 = 4x$  and  $x^2 = 4y$  by using method of integration.

**Answer:**

The given parabolas are

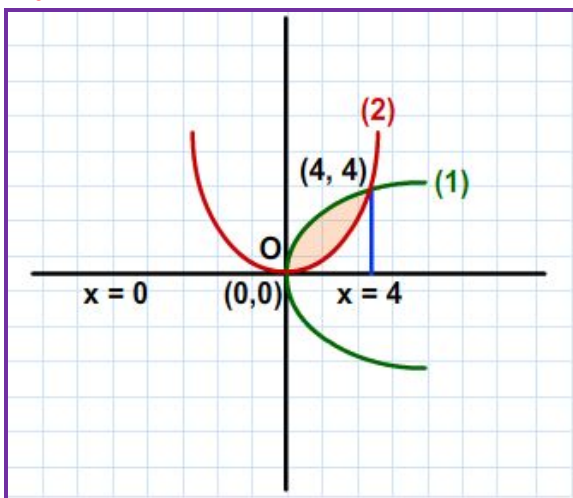
$$y^2 = 4x \quad \dots (i)$$

$$x^2 = 4y \quad \dots (ii)$$

(i) and (ii) meet at (0,0) and (4,4) area of the required region



$$= \int_0^4 (y_1 - y_2) dx$$



$$= \int_0^4 \left( \sqrt{4x} - \frac{x^2}{4} \right) dx$$

$$= \left[ 2 \cdot \frac{\frac{x^3}{2}}{\frac{3}{2}} - \frac{1}{4} \cdot \frac{x^3}{3} \right]_0^4$$

$$= \frac{4}{3} \cdot 8 - \frac{1}{12} \cdot 64 = \frac{16}{3} \text{ sq. units}$$

iii. If  $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ , show that  $A^2 = 5A + 7I$

**Answer:**

$$\Rightarrow A^2 = A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\therefore A^2 - 5A + 7I$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} + \begin{bmatrix} -15 & -5 \\ +5 & -10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 5-5+0 & 8-15+7 \\ -5+5+0 & 3-10+7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$



iv. Using the property of determinants and without expanding, show that

$$\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$$

**Answer:**

$$\begin{aligned} \text{LHS} &= \begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = \begin{vmatrix} 2 & 7 & 63+2 \\ 3 & 8 & 72+3 \\ 5 & 9 & 81+5 \end{vmatrix} \\ &= \begin{vmatrix} 2 & 7 & 63 \\ 3 & 8 & 72 \\ 5 & 9 & 81 \end{vmatrix} + \begin{vmatrix} 2 & 7 & 2 \\ 3 & 8 & 3 \\ 5 & 9 & 5 \end{vmatrix} \\ &= \begin{vmatrix} 2 & 7 & 7 \\ 3 & 8 & 8 \\ 5 & 9 & 9 \end{vmatrix} + 0 \end{aligned}$$

c.

- i. An oil company requires 13,000, 20,000 and 15,000 barrels of high grade, medium grade and low grade oil respectively. Refinery A produces 100, 300 and 200 barrels per day of high, medium and low grade oil respectively whereas the Refinery B produces 200, 400 and 100 barrels per day of high, medium and low grade oil respectively. If A costs Rs. 400 per day and B costs Rs. 300 per day to operate, how many days should each be run to minimize the cost of requirement?

**Answer:**

Let refineries A and B run for respectively x and y days. By hypothesis, we have the L.P.P as  
Minimise:

$$c = 400x + 300y$$

subject to the constraints

$$\begin{aligned} 100x + 200y &\geq 13000 & x + 2y &\geq 130 \\ 300x + 400y &\geq 20000 & \Leftrightarrow 3x + 4y &\geq 200 \\ 200x + 100y &\geq 15000 & 2x + y &\geq 150 \\ x \geq 0, y \geq 0 & & x \geq 0, y \geq 0. \end{aligned}$$

Now draw the lines :

$$x + 2y = 130 \quad \dots(1)$$

$$3x + 4y = 200 \quad \dots(2)$$

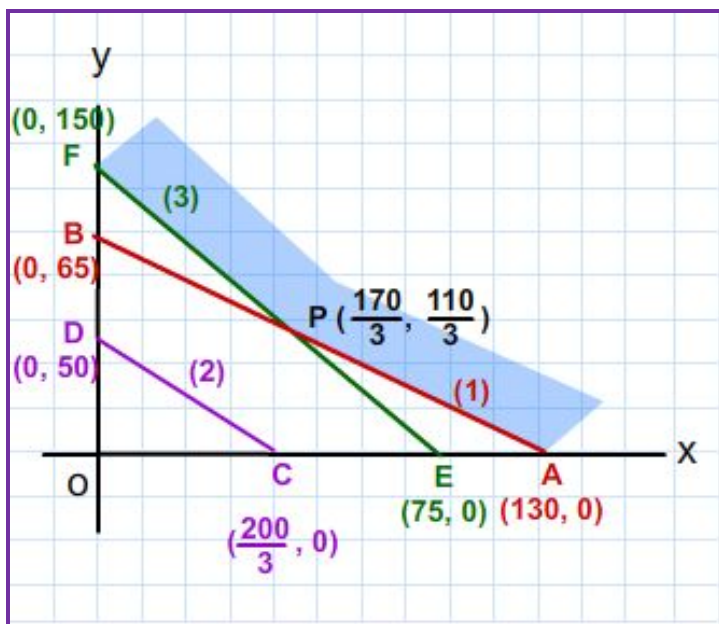
$$2x + y = 150 \quad \dots(3)$$

(1) and (3) meet at

$$P\left(\frac{170}{3}, \frac{110}{3}\right). \text{ The feasible region is}$$

APF (unbounded) and it is shaded.





Now value of  $c = 400x + 300y$

at  $A(130, 0)$  is 52000

at  $F(0, 150)$  is 45000

at  $P\left(\frac{170}{3}, \frac{110}{3}\right)$  is  $\frac{101000}{3} = 33666.67$

$\therefore$  cost is min. at  $P\left(\frac{170}{3}, \frac{110}{3}\right)$

$\Rightarrow$  Refinery A should run for  $\frac{170}{3}$  days and

Refinery B should run for  $\frac{110}{3}$  days

ii. Solve the differential equation:  $(1+x^2) \frac{dy}{dx} - 2xy = (x^2+2)(x^2+1)$ .

**Answer:**

We can write the given D.E. as  $\frac{dy}{dx} - \frac{2x}{1+x^2}y = x^2+2$ . (1)

Is a linear D.E. On comparing by  $\frac{dy}{dx} + Py = Q$

Here,  $P = -\frac{2x}{1+x^2}$ ,  $Q = x^2+2 \Rightarrow \int P dx = -\int \frac{2x}{1+x^2} dx = -\log(1+x^2) = \log(1+x^2)^{-1}$

I.F.  $= e^{\int P dx} = e^{\log(1+x^2)^{-1}} = (1+x^2)^{-1} = \frac{1}{1+x^2}$ .



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Here the sol. of (1) is  $y \cdot \frac{1}{1+x^2} = \int \frac{x^2+2}{x^2+1} dx + c = \int \left[ 1 + \frac{1}{x^2+1} \right] dx + c = x + \tan^{-1}x + c$

$$\Rightarrow y = (1+x^2)(x + \tan^{-1}x + c).$$

